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**Fundamentals and application of stimulated Brillouin scattering in optical fibre**

**Final Report**  
**(Contract Duration May 03-May 04)**

**by**  
**R. G. Harrison and W. Lu**  
**(May, 2004)**

***European Office of Aerospace Research and Development -EOARD***

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# Statement of Report

## I. Introduction

Current models of stimulated Brillouin scattering (SBS) with inclusion of spontaneous Brillouin scattering seeding show the Stokes signal to exhibit 100% modulation with no evidence of DC emission [1]. This is contrary to our experimental observations in which a DC contribution persists from the onset and increases with pump power up to more than 50% of the total Stokes signal [2]. In the first phase of this contract we have investigated this issue through re-evaluating spontaneous Brillouin scattering seeding in the theoretical model and comparing the new simulation results with existing experimental finding. We show that the DC component in the SBS emission may only be accounted for within the framework of this classical model by including a weak DC signal in the Gaussian white noise source that initiates the scattering process.

In the second phase of work we have extended this study to include the effects of feedback caused by the natural reflectivity at both ends of the SBS medium. While it is well known that for such conditions the Stokes signal exhibits rich forms of deterministic dynamics [3-6], a fact largely overlooked in the literature is that such dynamics is, as for the cavity-less case, also accompanied by an underlying DC component. Our simulations show that while this model can capture both the dynamics and DC features of the Stokes signal there exist a noticeable discrepancy in the magnitude of DC components. Further, the characteristics of the dynamics and DC signals are essentially independent of the inclusion, or not, of the DC component in the noise source.

In the third phase of this work we model an intermediate situation of very weak optical feedback, such that the Stokes signal may undergo transitions from stochastic to deterministic behaviour. Our recent experiments have shown that for such conditions, on increase of the pump power, a periodic signal grows from and subsides into the stochastic scattered emission, dependent on the strength of the emission and with a period characteristic of the system. These findings may be first evidence of Coherence Resonance (CR) in stimulated scattering. Our numerical simulations show qualitative agreement with these observations.

Our experimental results to which this report refers were all obtained using an experimental arrangement typical of that used in investigations of SBS in optical fibre, pumped by a single longitudinal and fundamental transverse mode continuous wave Nd:YAG laser.[6] Single mode fibre was used throughout. This was cleaved off-axis to minimise cavity feedback except for investigations of SBS with cavity feedback for which the fibre was cleaved normal to its axis

## II. Results on SBS

### 1) SBS Model

Stimulated Brillouin scattering is described as a three-wave parametric coupling process between two electromagnetic waves, the pump and Stokes, and an acoustic wave. With the inclusion of noise initiation, the model description in a normalized form becomes [6]

$$\begin{aligned}\frac{\partial A}{\partial \eta} + \frac{\partial A}{\partial \xi} + \frac{1}{2}\beta A &= -gBC, \\ \frac{\partial B}{\partial \eta} - \frac{\partial B}{\partial \xi} + \frac{1}{2}\beta B &= gAC^*, \\ \frac{1}{\beta_A} \frac{\partial C}{\partial \eta} + C &= AB^* + f(\eta, \xi),\end{aligned}$$

where  $A$ ,  $B$  and  $C$  are respectively the slowly varying amplitudes of the forward-propagating pump, backward-scattered Stokes and the acoustic wave, all normalized to the incident pump amplitude  $A_0$  at  $z = 0$ .

$f(\eta, \xi)$  is the Gaussian white noise source describing the random thermal fluctuation of the density of the medium.  $\eta = t/t_r$  and  $\xi = z/L$  are normalized time and space coordinates and  $t_r = nL/c$  is the propagating time of the light waves in the fiber of the length  $L$ .  $\beta = \alpha L$  and  $\beta_A = \pi \Delta \nu_B t_r$  are the normalized power loss of the light waves and the relaxation rate of sound in the medium, where  $\alpha$  and  $\Delta \nu_B$  are the loss coefficient and the spontaneous linewidth, respectively. This model description was set in the seminal works by Kroll and Tang in the mid-sixties and is that appearing in standard texts.

## 2) Simulation results and experimental comparison

In our simulation,  $f(\eta, \xi)$  comprises two parts, a Gaussian white noise term ( $\delta$  correlated and zero average) and a relatively weak constant noise term, the latter being set as  $\sim 1\%$  of the amplitude of the former. Fig.1 (a) shows the characteristic of the Stokes intensity as a function of the pump strength, both are normalised. As seen, both the total signal (AC+DC) and the DC component are linearly dependent on the pump strength. The proportion of DC part in the total signal is shown to increase on increasing the pump strength, for example  $\sim 50\%$  around 2-3 times of the SBS threshold and  $\sim 60\%$  at 6 times of threshold. The DC component is also shown in the transmitted pump, as displayed in Fig.1(b), in which the DC proportion also increase with the pump strength. Comparison of these simulation results with our experimental findings in Fig 2 show qualitatively good agreement. More generally we find that in simulation the relative proportion of DC to AC is dependent on the DC level of spontaneous emission. We are presently seeking a physical justification and interpretation for inclusion of the DC component in the noise initiation. This may stem from a phenomenon of spectral self phase conjugation as discussed below (see Section V).

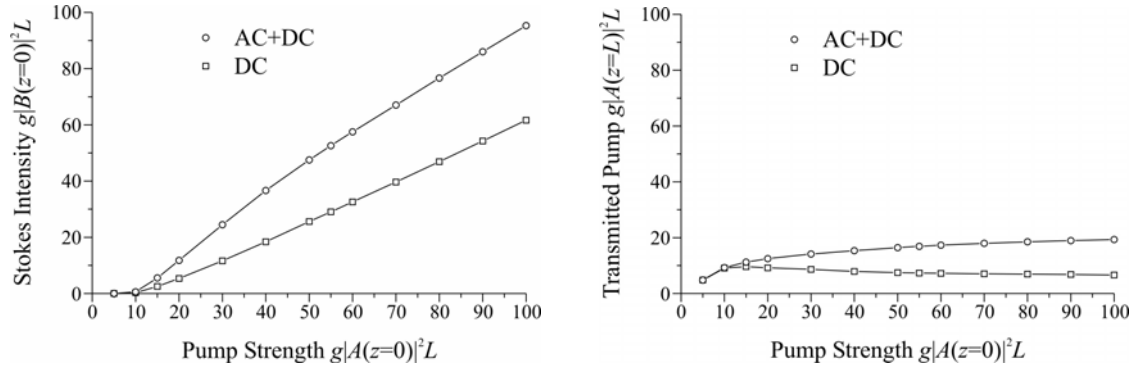


Fig.1: (a) Transmitted pump and (b) scattered Stokes intensities as a function of the incident pump strength for  $\beta_A = 148$  ( $L = 290$  m,  $\delta_e = 1.0$  or  $L = 124$  m,  $\delta_e = 2.36$ ).

## Relation of Noise Intensity to Pump Intensity

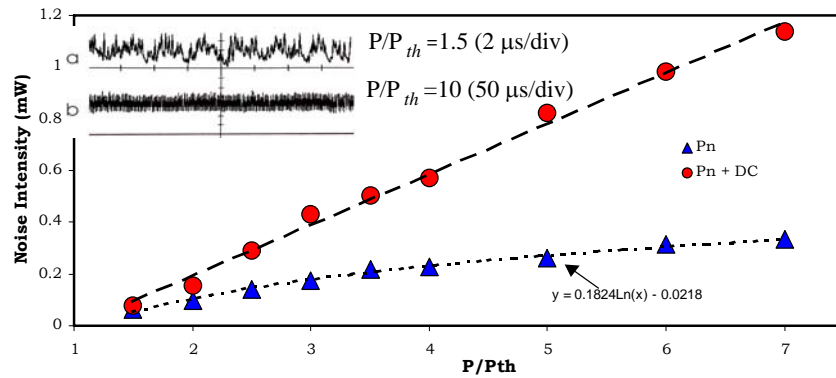


Fig.2: Experimental measurement of the total Stokes emission (circles) and the noise component (triangles) as a function of the pump strength normalized to its threshold value.

## III. Results on SBS with cavity

### 1) SBS Model in the presence of cavity

With the inclusion of noise initiation and weak feedback, An extra equation is required to describe the slowly varying amplitudes of the forward- scattered Stokes due to reflection

$$\frac{\partial D}{\partial \eta} + \frac{\partial D}{\partial \xi} + \frac{1}{2}\beta D = 2iu(|A|^2 + |B|^2)D,$$

For the case of weak feedback ( $R_1 R_2 \ll 1$ ) the above equations are added with the following boundary conditions:

$$D(\eta, \xi = 0) = \sqrt{R_1} B(\eta, \xi = 0),$$

$$B(\eta, \xi = 1) = \sqrt{R_2} D(\eta, \xi = 1).$$

In this model description the spectral line broadening is taken to be homogeneous, as in bulk media. Due to the enhancements in the fibre, however, the bandwidth  $\Delta\nu_B$  can be enlarged by a factor  $\delta_e$  of 1–2.5, i.e.,  $\Delta\nu_B \equiv \delta_e \Delta\nu_B$ . We note that the introduction of  $\delta_e$  is in fact equivalent to the increase of the length  $L$  in  $\delta_e$  times.

## 2) Simulation results in the presence of cavity and comparisons with experimental findings

Let us first investigate the characteristics of the SBS output in the case that  $f(\eta, \xi)$  is a pure Gaussian white noise ( $\delta$ -correlated and zero averaged). Figures 3 and 4 show the typical dynamics of SBS emission for increasing pump power (the gain parameter  $g$ ) for two different fibre lengths. As seen from Figs. 3(a) and 4(a), a 100% SBS modulation with the period  $2t_r$  is observed from the onset of SBS lasing (for  $R_1 = R_2 \equiv R = 3.5\%$ , the SBS threshold  $g_{th} \cong 3.4$ ). Increasing the pump strength leads to the appearance of DC component in the SBS intensity [Figs. 3(b) and 4(b)]; the period of oscillations is  $2t_r$ . For the short fibre length, a further increase of the pump strength results in gradual decrease of the modulation amplitude (AC component) and increase of the DC background, leading to a pure DC output [Fig. 3(c)]. For the longer fibre length, however, the dynamics is shown to be richer: the SBS signal may experience a bifurcation to a periodic oscillation state with the period equal  $t_r$  [Fig. 4(c)] before the pure DC is observed [Fig. 4(d)]. We have further shown more oscillation states with different periods for even longer fibre length. Comparisons of these simulation results with our experimental findings show a qualitatively good agreement in terms of dynamics of SBS output.

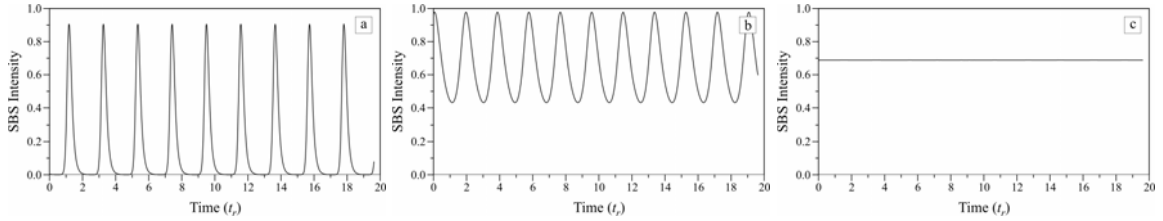


Fig. 3: Time series of SBS intensities for  $u = 0$ ,  $R = 3.5\%$  and  $\beta_A = 33$  ( $L = 65$  m,  $\delta_e = 1.0$ ). (a)  $g = 4$ . (b)  $g = 9$ . (c)  $g = 10$ .

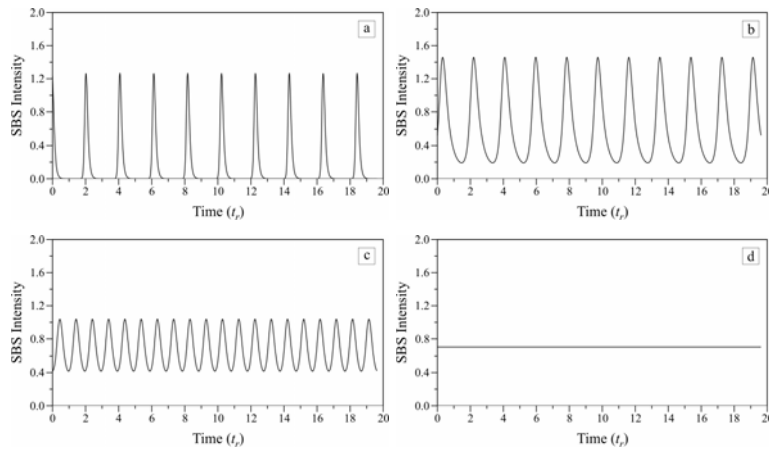


Fig. 4: Time series of SBS intensities for  $u = 0$ ,  $R = 3.5\%$  and  $\beta_A = 50$  ( $L = 98$  m,  $\delta_e = 1.0$ ). (a)  $g = 4$ . (b)  $g = 8$ . (c)  $g = 10$ . (d)  $g = 11$ .

However, our simulation results cannot fit well with the experiment when the proportion of DC and AC components in the SBS emission are compared. Figure 5 shows the experimental (solid squares) and theoretical (solid circles) measurements of the DC component normalized by the total Stokes emission (DC+AC) as functions of the pump strength. As seen, this experimentally measured ratio is (almost) linearly dependent on the pump strength from the onset of SBS emission until it reaches to unity, which implies 100% DC SBS output. On the contrary, the theoretical results show that the build-up of the DC component is much slower on increasing the pump strength (open circles) in the window between  $1 < P/P_{th} < 2$ . As a result, this curve is shifted to the right along the  $P/P_{th}$ -axis.

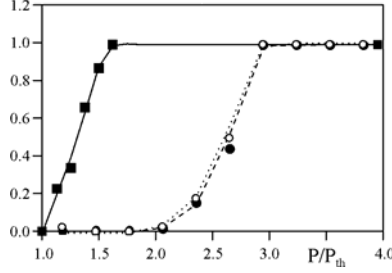


Fig. 5: Experimental (squares) and theoretical (circles) measurements of the DC component normalized by the total Stokes emission (DC+AC) as functions of the pump strength normalized to its threshold value for  $u = 0$ ,  $R = 3.5\%$  and  $\beta_A = 33$  ( $L = 65$  m,  $\delta_e = 1.0$ ). Black and white circles correspond to constant noise term equal 0 and 5% of the white noise term amplitude.

Building on our results obtained above (see Section II), we further consider that the noise initiation  $f(\eta, \xi)$  comprises two parts, a Gaussian white noise term ( $\delta$ -correlated and zero averaged) and a constant noise term. The ratio DC/(AC+DC) with the inclusion of DC component in the spontaneous SBS emission is shown in Fig.5 as the open circles on increasing the pump strength. As seen, the difference of the ratios in the cases of with and without a constant noise term is very small; it is so even for relatively large constant seeding. These observations show that the relative proportion of DC to AC is generally not dependent on the DC level of spontaneous emission in the presence of an external cavity. This is perhaps reasonable since, with the external cavity, the fibre system operates as a laser system, rather than a nonlinear amplifier of spontaneous emission as in the cavity-less case discussed above (Sect. II).

#### IV Coherence resonance (CS)

##### 1) CS phenomenon

The paradoxical effect of noise induced ordering of the temporal behaviour of nonlinear systems has been a subject of keen interest for upwards of two decades. In contrast with linear systems, for which noise normally reduces their regular output characteristics, in nonlinear systems the effect of noise may result in increased coherence, amplification, signal-to-noise ratio as well as new more ordered regimes or structures. One such phenomenon is Stochastic Resonance (SR), which has been extensively researched. Recently a new phenomenon related to SR is Coherence Resonance (CR), which was first considered to be SR without an input periodic signal, in a study on the effect of noise on a limit cycle near a bifurcation point. Later it became better characterised in its own right [7], and has since been theoretically predicted in diverse systems including a Fitz Hugh-Nagumo neuron and pancreatic cells and experimentally observed in a laser diode with optical feedback and a monovibrator circuit. Here we report first experimental evidence of CR in stimulated scattering, specifically stimulated Brillouin scattering (SBS) excited in an optical fibre with very weak feedback by a continuous wave (CW) laser signal. Our findings, which occur well below the threshold for SBS lasing, show the scattered Stokes signal displays stochastic dynamical behaviour from its onset. On increase of the pump power a periodic signal grows from and subsides into the stochastic scattered emission, dependent on the strength of the emission and with a period characteristic of the system, here defined by the length of the fibre. We show that the classical three-wave model description of SBS with feedback, described above (Section III), can explain qualitatively some of the key features of CS.

##### 2) Experimental observation of CS

A representative example of our findings is shown in Fig.6 which show time recordings of the dynamical component of the Stokes signal power for three pump levels, taken using a fibre of length  $L=240$  meters. As

discussed in Sect. II the Stokes signal, for the conditions of negligible feedback used here, comprised a dynamical signal superimposed on a DC component the relative magnitude of which increased with pump strength. These recordings do not show the CW component. For a pump power near the SBS threshold the Stokes signal displays irregular behaviour (Fig.6a). On increasing the pump power, the signal shows evidence of underlying periodicity which grows to become the dominant feature in the Stokes signal (Fig.6b) and then decreases for further increase of pump power to finally revert to aperiodicity (Fig.6c). The period of the oscillatory emission is measured to be  $\sim 2.3 \mu\text{sec}$ , which corresponds to the round trip time for light in the fibre,  $t_r = 2nL/c$  with a corresponding frequency  $\omega_0 = 430 \text{ kHz}$  and harmonics, clearly evident in the power spectra.

In order to quantify the coherency of the system, we use the measure of coherence  $\beta$ , which has also been interpreted as the signal-to-noise ratio.  $\beta$  is a convenient measure to use as it is robust enough for statistical measures with the amount of data that we have, and can be calculated as:

$$\beta = h_0 \left( \frac{\Delta\omega}{\omega_0} \right)^{-1}, \quad (1)$$

where  $\Delta\omega/\omega_0$  is the familiar quality factor  $Q$ ;  $\Delta\omega$  is the peak width at  $h_1$  ( $h_1 = e^{-1/2}h_0$  where  $h_0$  is the peak height of the normalised power spectra density) and  $\omega_0$  is the frequency of resonance. The  $\beta$  factor for the backscattered intensity was calculated for each dataset taken at seven different pump levels (see dots in Fig. 7). Calculating the mean and standard deviation of the Stokes signal at each pump level, we see a robust increase and then decrease in  $\beta$  at around  $0.175\text{W}$  as the laser power is increased, with the entire range of coherency ranging over 3 orders of magnitude. These results are similar in order of magnitude and shape to those reported by others in other systems.

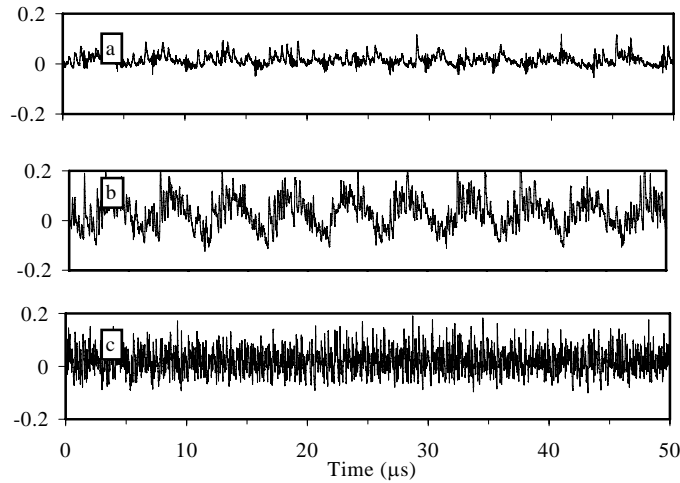


Figure 6. AC component of the Stokes signal at pump levels 2Pth (a), 4Pth (b), and 7Pth (c). The vertical axis is Stokes intensity for each pump level in relative units.

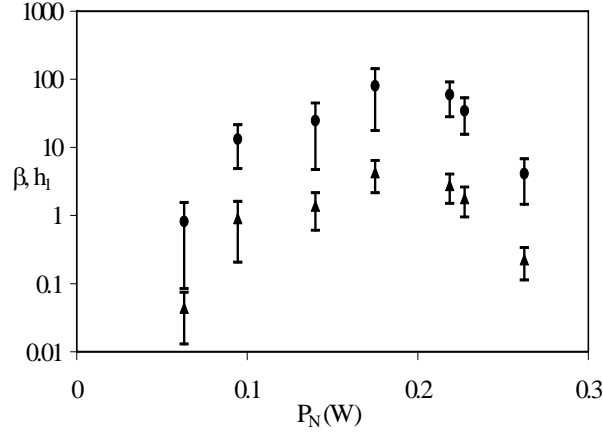
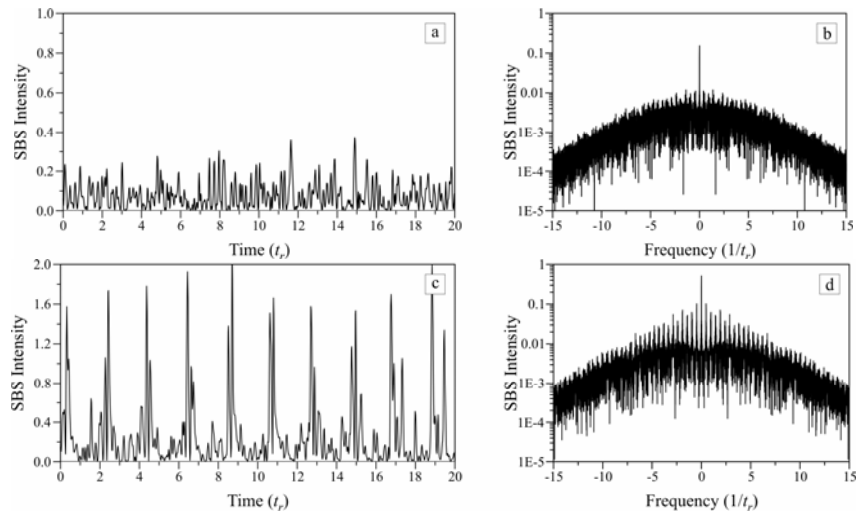


Figure 7:  $\beta$  and  $h_1$  versus SBS noise power  $P_N$ . Dots are mean  $\beta$ ; triangles are the mean of the peak height at  $h_1 = e^{-1/2}h_0$  of the normalised power spectral density. In both cases, the error bars are one standard deviation

### 3) Model simulation of CR

The SBS model we use here was fully described in Sect III. Figure 8 shows typical dynamics of SBS emission in the presence of a very weak cavity for increasing pump power (the gain parameter  $g$ ). As seen from Figs. 8(a, b), 100% SBS modulation is observed from the onset of SBS lasing (for  $R_1 = R_2 \equiv R = 5 \times 10^{-4}\%$ , the SBS threshold  $g_{th} = 10.485$ ). Increasing the pump strength leads to appearance of noticeable periodicity at  $1/2t_r$ ; the corresponding spectra of SBS intensity, however, indicates the existence of a number of equidistant frequencies  $1/2t_r$ ,  $1/t_r$ ,  $3/2t_r$  etc. [Figs. 8(c, d)]. With further increase of pump strength, the visual periodicity is absent [Figs. 6(e, g)], which is in good agreement with the experimental results shown in Fig. 6. Note Fig. 8 does not show the DC pedestal that has been discussed in the first report. As discussed in that report the coherent model as above cannot capture this feature unless an additional DC component is included into the noise term. As follows from Figs. 8 (f, h), this behaviour is due to relative strengthening of higher harmonics in the SBS emission. This feature is especially pronounced in Fig. 8(g): weakened low frequencies form a hole in the SBS spectrum. In other words, on increasing pump strength, we observe a competition of different spectral components (modes). We note that this feature was also observed in the natural fibre cavity with higher  $R$  as discussed in Sect III. The major difference for the latter case is the absence of noise in the SBS emission, the dynamics being purely deterministic.





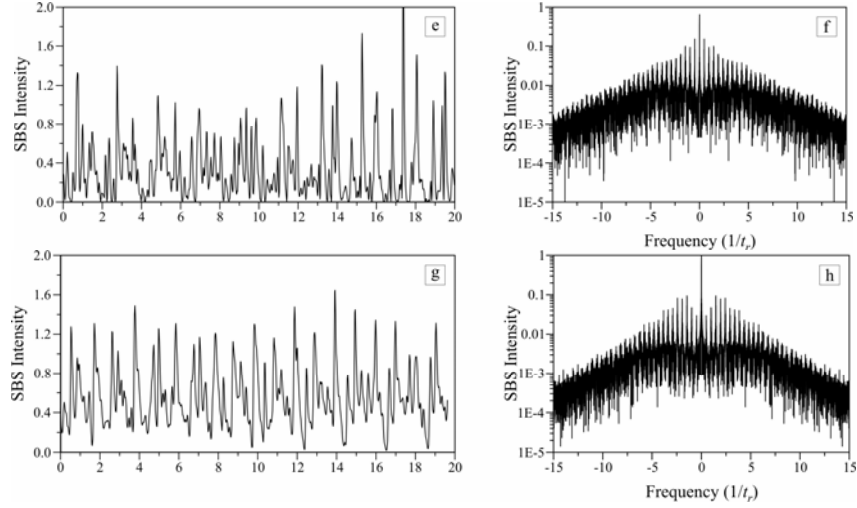


Fig. 8: Time series and corresponding spectra of SBS intensities for  $u = 0$ ,  $R = 5 \times 10^{-4}\%$  and  $\beta_A = 120$  ( $L = 236$  m,  $\delta_e = 1.0$ ). (a, b)  $g = 14$ . (c, d)  $g = 18$ . (e, f)  $g = 20$ . (g, h)  $g = 30$ .

## V. Discussion and Conclusions

The purpose of this programme of work has been to investigate and evaluate in detail the conventional (classical) model description of SBS. This re-examination has been motivated by apparent inconsistencies between the predictions of this model and recent experimental findings. This has become particularly apparent through using CW pumping in optical fibre to better define and clarify the characteristics of SBS than hitherto possible using the traditional pulsed excitation of much of the earlier work. Of these inconsistencies the most obvious is the occurrence of DC Stokes emission which is intuitively inconsistent with this model. We find that this feature may only be reproduced if we introduce within the seeding term in this model a small DC contribution over and above the conventional noise term. However, there is no obvious physical justification for this. In fact our most recent work [8] points to a new phenomenon of spectral self phase conjugation as explanation of DC Stokes emission which involves spectral broadening of the Stokes signal through that of the pump arising from gain saturation. However this process cannot be described by the classical model used above since the latter is a single mode treatment based on plane wave analysis in which each of the waves is considered to be monochromatic. In light of this our findings nevertheless suggest that there may exist a linkage between these two treatments.

Turning next to the influence of external feedback on the characteristics of the Stokes emission we have found as in earlier reports that the signal becomes predominantly deterministic exhibiting a dynamic and DC component the latter of which grows with pump strength to become the dominant contribution. Interestingly the relative proportion of DC to AC is generally not dependent on inclusion of a DC component in the seeding term (spontaneous emission term). This is perhaps reasonable since, with the external cavity, the fibre system operates as a laser system, rather than a nonlinear amplifier of spontaneous emission as in the cavity-less case discussed above (Sect. II). Consequently the mode selecting property of the cavity acts to promote the growth of those frequencies within the noise that are modes of the cavity, hence promoting deterministic behaviour over stochastic. As in conventional lasers it is this that dominates over the spectral form of the spontaneous emission; in SBS, whether the spontaneous noise contains a DC contribution or not. Our simulations show that while this model can capture both the dynamics and DC features of the Stokes signal there exist a noticeable discrepancy in the magnitude of DC components. In other recent work we have found that the spectrum of SBS is in fact inhomogeneous, [2,9] rather than homogeneous as in this classical model. The inhomogeneity derives from the ability of such media to guide a fan of beam directions, the Stokes spectrum then being a convolution of frequency shifted homogeneously broadened components each generated from different angular components of the pump and Stokes signals. This phenomenon may account for the above discrepancy. More importantly in view of its generic nature there is a pressing need for the development of SBS theory to account for this spectral broadening phenomenon.

Finally, in this work we have shown through numerical results that the classical model of SBS can qualitatively capture some of the characteristics we observe in the transition from stochastic to deterministic dynamics of the Stokes emission for conditions of very weak optical feedback. These findings may provide first evidence of the phenomenon of Coherence Resonance in stimulated scattering.

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